DOMAIN, RANGE, LIMIT OF FUNCTION (BM)

Find the Domain of the following Functions:

1. $y = \sqrt{2x - x^2}$
2. $y = \sqrt{1 - x^2}$
3. $y = \sqrt{x-1} + \sqrt{6-x}$
4. $y = \sqrt{\dfrac{x+3}{5-x}}$
5. $y = \sqrt{-4x^2 + 4x + 3}$
6. $y = \dfrac{1}{x-1} + \sqrt{2+x}$
7. $f(x) = \dfrac{\sqrt{12 + x - x^2}}{x(x-2)}$
8. $y = \sqrt{\dfrac{x^2 - 5x + 6}{x^2 + 6x + 8}}$
9. $y = \dfrac{1}{3 - \log_3(x-3)}$
10. $y = \dfrac{\sqrt{x+5}}{\log_e(9-x)}$
11. $y = \log_2 \dfrac{x-2}{x+2}$
12. $y = \sqrt{\log_e(x+1)}$
13. $y = \sqrt{\log_e \dfrac{3-x}{x}}$
14. $y = \sqrt{4x - x^2} + \log_e(x^2 - 1)$
15. $y = \sqrt{\log_{0.3}(x^2 - 5x + 7)}$

Find range of the following Functions:

16. x^2
17. $|x| + 1$
18. $|\sin x| - 2$
19. $2\cos^2 x + \sin^2 x + 1$
20. $x^2 + 2x$
21. $2x^2 + 6x + 19$
22. $x^2 - 3x + 4$
23. $\sqrt{9 - x^2}$
24. $x^4 + 4x^2 + 10$
25. $2\cos x + 3$
26. $\dfrac{1}{5 + 3\cos x}$
27. $\dfrac{2}{1 + x^2}$
28. $\dfrac{x^2}{1 + x^2}$
29. $\dfrac{x^2 - 1}{x - 1}$
30. $|x - 1| + |x - 2|,\ -1 \le x \le 3$
31. $\dfrac{1}{x}$
32. $\dfrac{1}{x},\ x \in [-2,\ 3] - \{0\}$
33. $x^2 + 1,\ x \in [-3,\ 2]$
34. $2 - \dfrac{1}{x},\ x \in [-5,\ -1]$
35. $f(x) = x^2 + \dfrac{1}{x^2}$. Also find x for $f(x) = f_{min}$
36. $\dfrac{1}{2x - 3},\ x \in [1,\ 5]$
37. $4^x - 2^x + 1$
38. $4^x + 2^x + 1$
39. $\log_{\sqrt{7}}(3\cos x + 4)$
40. $3\sin\sqrt{\dfrac{\pi^2}{9} - x^2}$

Find the following limits:

41. $\lim\limits_{x \to 1} \dfrac{4x^5 + 9x + 7}{3x^6 + x^3 + 1}$
42. $\lim\limits_{x \to 1} \dfrac{x^2 - 3x + 2}{x - 1}$
43. $\lim\limits_{x \to 2} \dfrac{x^5 - 32}{x^3 - 8}$
44. $\lim\limits_{x \to 0} \dfrac{(4 + x)^3 - 64}{x}$
45. $\lim\limits_{x \to 6} \dfrac{x - 6}{\sqrt{x+3} - 3}$
46. $\lim\limits_{x \to 0} \dfrac{\sin 2x}{x}$
47. $\lim\limits_{x \to 0} \dfrac{\sin(x^2)}{x}$
48. $\lim\limits_{x \to 0} \dfrac{1 - \cos x}{x^2}$
49. $\lim\limits_{x \to 0} \dfrac{\sin 3x}{\sin 2x}$

50. $\lim_{x\to0}\frac{\sin4x-\sin2x}{x}$

51. $\lim_{x\to1}\frac{\sin(2x^2-x-1)}{(x-1)}$

52. $\lim_{x\to0}\frac{\tan3x}{\sin5x}$

53. $\lim_{x\to0}\frac{e^{5x}-1}{x}$

54. $\lim_{x\to0}\frac{\log(1+4x)}{x}$

55. $\lim_{x\to0}\frac{\log(1+x^2)}{x}$

56. $\lim_{x\to0}\frac{e^{x^2}-1}{\sin x}$

57. $\lim_{x\to0}\frac{\log(1+3x)}{\sin2x}$

58. $\lim_{x\to0}\frac{5^x-3^x}{x}$

59. $\lim_{x\to0}\frac{\log(1+x)-\log(1-x)}{x}$

60. $\lim_{x\to0}\frac{\sin x-\tan x}{\sin^3 x}$

ANSWER KEY

1. D(y) = [0, 2]
2. D(y) = [–1, 1]
3. D(y) = [1, 6]
4. D(y) = [3, 5]
5. D(y) = [–1/2, 3/2]
6. D(y) = [–2, 1) ∪ (1, + ∞)
7. [–3, 0) ∪ (0, 2) ∪ (2, 4]
8. (–∞, –4)∪(–2, 2]∪[3, + ∞)
9. (3, 30) ∪ (30, + ∞)
10. D(y) = [–5, 8) ∪ (8, 9)
11. (–∞, –2) ∪(2, + ∞)
12. D(y) = [0, + ∞)
13. D(y) = (0, 3/2]
14. D(y) = (1, 4]
15. D(y) = [2, 3]
16. [0, ∞)
17. [1, ∞)
18. [–2, –1]
19. [2, 3]
20. [–1, ∞)
21. $\left[\frac{29}{2},\ \infty\right)$
22. $\left[\frac{7}{4},\ \infty\right)$
23. [0, 3]
24. [10, ∞)
25. [1, 5]
26. $\left[\frac{1}{8},\ \frac{1}{2}\right]$
27. (0, 2]
28. [0, 1]
29. R – {2}
30. [1, 5]
31. R – {0}
32. $\left(-\infty,\ -\frac{1}{2}\right]\cup\left[\frac{1}{3},\ \infty\right)$
33. [1, 10]
34. $\left[\frac{9}{5},\ 1\right]$
35. [2, ∞) min at x = ±1
36. $(-\infty,\ -1]\cup\left[\frac{1}{7},\ \infty\right)$
37. $\left[\frac{3}{4},\ \infty\right)$
38. (1, ∞)
39. [0, 2]
40. $\left[0,\ \frac{3\sqrt{3}}{2}\right]$
41. 4
42. –1
43. $\frac{20}{3}$
44. 48
45. 6
46. 2
47. 0
48. $\frac{1}{2}$
49. $\frac{3}{2}$
50. 2
51. 3
52. $\frac{3}{5}$
53. 5
54. 4
55. 0
56. 0
57. $\frac{3}{2}$
58. $\log\left(\frac{5}{3}\right)$
59. 2
60. $-\frac{1}{2}$